BARI-TH/05-510

Radiative transitions of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$

P. Colangelo, F. De Fazio and A. Ozpineci

Istituto Nazionale di Fisica Nucleare,

Sezione di Bari, Italy

Abstract

We study radiative decays of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ using light-cone QCD sum rules. In particular, we consider the decay modes $D^*_{sJ}(2317) \to D^*_s \gamma$ and $D_{sJ}(2460) \to D^{(*)}_s \gamma, D^*_{sJ}(2317)\gamma$ and evaluate the hadronic parameters in the transition amplitudes analyzing correlation functions of scalar, pseudoscalar, vector and axial-vector quark currents. In the case of $D^*_{sJ}(2317) \to D^*_s \gamma$ we also consider determinations based on two different correlation functions in HQET. The decay widths turn out to be different than previous estimates obtained by other methods; the results favour the interpretation of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ as ordinary $\bar{c}s$ mesons.

PACS numbers: 12.38.Lg, 13.20.Fc

I. INTRODUCTION

The observation of two narrow resonances with charm and strangeness, $D^*_{sJ}(2317)$ in the $D_s\pi^0$ invariant mass distribution [1, 2, 3, 4, 5, 6, 7] and $D_{sJ}(2460)$ in the $D^*_s\pi^0$ and $D_s\gamma$ mass distributions [2, 3, 4, 7, 8, 9], has raised discussions about the nature of these states and their quark content [10]. The natural identification consists in considering these states as the scalar and axial vector $\bar{c}s$ mesons respectively denoted as D_{s0} and D'_{s1}. In the heavy quark limit $m_c \to \infty$ such states are expected to be degenerate in mass and to form a doublet having $s_\ell^P = \frac{1}{2}^+$, with s_ℓ the angular momentum of the light degrees of freedom. In that interpretation the two mesons complete, together with $D_{s1}(2536)$ and $D_{s2}(2573)$, the set of four states corresponding to the lowest lying P-wave $\bar{c}s$ states of the constituent quark model. A chiral symmetry between the negative and positive parity doublets $D_s - D^*_s$ *vs* $D_{s0} - D'_{s1}$, suggested in ref.[11, 12], would account for the equality of the hyperfine splitting in the two doublets.

However, estimates of the masses of these mesons based on potential quark models generally produce larger values than the measured ones, implying that the two scalar and axial-vector $\bar{c}s$ D_{s0} and D'_{s1} states should be heavy enough to decay to DK and D^*K and should have a broad width. On this basis, other interpretations for $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ have been proposed, for example that of molecular states [13]. Unitarity effects in the scalar DK channel have also been considered [14].

Radiative transitions probe the structure of hadrons, and therefore they are suitable to understand the nature of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ distinguishing among different interpretations [15, 16]. Their rates can be predicted by various methods and the predictions can be compared to the experimental measurements. In particular, it has been suggested that, in the molecular picture, the $D_{sJ}(2460) \to D^*_{sJ}(2317)\gamma$ decay should be driven by the $D^* \to D\gamma$ transition and should occur at a different rate with respect to the rate for a quark-antiquark meson decay [13]; such a suggestion has to be supported by explicit calculations in view of the experimental observations.

The radiative decay widths $D^*_{sJ}(2317) \to D^*_s\gamma$ and $D_{sJ}(2460) \to D_s^{(*)}\gamma, D^*_{sJ}(2317)\gamma$ have been evaluated using the constituent quark model [11, 15] and the Vector Meson Dominance (VMD) ansatz in the heavy quark limit [10, 16]. In this paper we use a different method, light-cone QCD sum rules, an approach exploited to analyze many aspects of the heavy and

light quark system phenomenology [17, 18], including radiative decays [19, 20] (for a review and references see [21]). We apply the method starting from the identification of $D^*_{sJ}(2317)$ with D_{s0} and $D_{sJ}(2460)$ with D'_{s1} and we discuss results and related uncertainties. In particular, in Section II we consider the decay mode $D^*_{sJ}(2317) \to D^*_s\gamma$ and describe in detail the calculation of the transition amplitude, the input quantities used in the analysis, the numerical results and the sources of uncertainties. In Section III we carry out a calculation of the same transition amplitude in the infinite heavy quark limit, discussing the deviation from the case of finite mass which is sizeable in the case of charm. The radiative modes of $D_{sJ}(2460)$ are analyzed in Sections IV-VI, where we find different results with respect to those obtained by other methods. In Section VII we discuss the differences; in spite of them, considering the available experimental measurements, we conclude that the description of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ as $q\bar{q}$ states is favoured.

II. $D^*_{sJ}(2317) \to D^*_s\gamma$

The amplitude of the E1 transition $D_{s0} \to D^*_s\gamma$:

$$\langle\gamma(q,\lambda)D^*_s(p,\lambda')|D_{s0}(p+q)\rangle = ed\,[(\varepsilon^*\cdot\tilde{\eta}^*)(p\cdot q) - (\varepsilon^*\cdot p)(\tilde{\eta}^*\cdot q)] \quad , \tag{2.1}$$

with $\varepsilon(\lambda)$ and $\tilde{\eta}(\lambda')$ the photon and D^*_s polarization vectors, respectively, and e the electric charge, involves the hadronic parameter d which has dimension mass^{-1}. According to the strategy of QCD sum rules, the calculation of this parameter starts from considering the QCD and the hadronic expressions of a suitable correlation function of quark currents.

We consider the correlation function

$$F_\mu(p,q) = i\int d^4x\, e^{ip\cdot x}\langle\gamma(q,\lambda)|T[J^\dagger_\mu(x)J_0(0)]|0\rangle \tag{2.2}$$

of the scalar $J_0 = \bar{c}s$ and the vector $J_\mu = \bar{c}\gamma_\mu s$ quark currents, and an external photon state of momentum q and helicity λ. The correlation function can be expressed in terms of Lorentz invariant structures:

$$F_\mu(p,q) = F_0\ \left[(p\cdot\varepsilon^*)q_\mu - (p\cdot q)\varepsilon^*_\mu\right] + \ldots \quad . \tag{2.3}$$

In order to compute F_0 (or F_μ) in QCD, we carry out the light-cone expansion $x^2 \to 0$ of the product of the two currents in (2.2). This involves non-local matrix elements of quark

operators between the vacuum and the photon state which can be expressed in terms of operator matrix elements of increasing twist. For example, contracting the charm-quark fields in eq.(2.2) we obtain

$$F_\mu(p,q) = \int \frac{d^4k}{(2\pi)^4} \int d^4x \, \frac{e^{i(p-k)\cdot x}}{m_c^2 - k^2} \langle \gamma(q,\lambda)|\bar{s}(x)\gamma_\mu(\not{k} + m_c)s(0)|0\rangle$$

$$= \int \frac{d^4k}{(2\pi)^4} \int d^4x \, \frac{e^{i(p-k)\cdot x}}{m_c^2 - k^2} \Big[k_\mu \langle \gamma(q,\lambda)|\bar{s}(x)s(0)|0\rangle$$

$$- \; ik^\alpha \langle \gamma(q,\lambda)|\bar{s}(x)\sigma_{\mu\alpha}s(0)|0\rangle + m_c \langle \gamma(q,\lambda)|\bar{s}(x)\gamma_\mu s(0)|0\rangle \Big]; \qquad (2.4)$$

the expressions of the photon matrix elements in terms of distribution amplitudes are collected in the Appendix. This kind of contributions is depicted in fig.1(a). Moreover, the light-cone expansion involves higher-twist contributions related to three-particle quark-gluon matrix elements, as depicted in fig.1(b); the expressions of the relevant quark-gluon matrix elements can also be found in the Appendix.

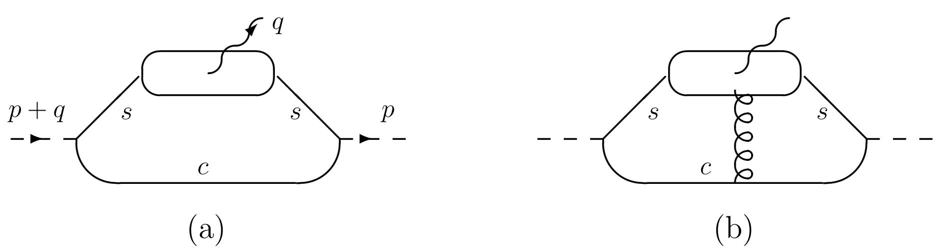

FIG. 1: Diagrams involving photon distribution amplitudes. The dashed lines represent the two currents in the correlation function (2.2). In (a) two-particle contributions and in (b) three-particle quark-gluon contributions are shown.

In addition to the contributions of the photon emission from the soft s quark, we must consider the perturbative photon coupling to the strange and charm quarks, fig.2 (a) and (b). It produces an expression for F_0 of the form:

$$F_0 = \int_{(m_s+m_c)^2}^{+\infty} ds \frac{\rho^P(s)}{(s-p^2)(s-(p+q)^2)} \qquad (2.5)$$

with

$$\rho^P(s) = -\frac{3e_s}{4\pi^2}\left\{-m_s \ln\left(\frac{s-m_c^2+m_s^2-\lambda^{\frac{1}{2}}(s,m_c^2,m_s^2)}{s-m_c^2+m_s^2+\lambda^{\frac{1}{2}}(s,m_c^2,m_s^2)}\right)+\frac{m_c-m_s}{s}\lambda^{\frac{1}{2}}(s,m_c^2,m_s^2)\right\}$$

$$+\ \frac{3e_s}{4\pi^2}\frac{m_c+m_s}{2}\frac{\lambda^{\frac{1}{2}}(s,m_c^2,m_s^2)}{s}\left(1-\frac{m_s^2-m_c^2}{s}\right)\ +\ (s\leftrightarrow c) \qquad (2.6)$$

(λ the triangular function). Furthermore, nonperturbative effects when the photon is emitted from the heavy quark give rise to contributions proportional to the strange quark condensate, corresponding to the diagram in fig.2 (c).

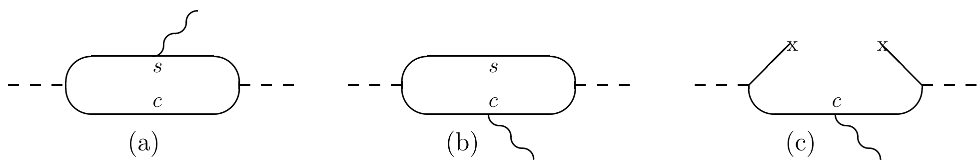

FIG. 2: Perturbative photon emission by the strange (a) and charm (b) quark. In (c) the strange quark condensate contribution is represented.

The result is an expression of the correlation function (2.2) and of the function F_0 in terms of quantities such as quark masses, condensates and photon distribution amplitudes. The sum rule for d is obtained by the equality of this QCD expression with a hadronic expression obtainted by a complete insertion of physical states. The two quark currents in (2.2) have non-vanishing matrix elements between the vacuum and D_s^* and D_{s0}:

$$\langle 0|J_\mu^\dagger|D_s^*\rangle = f_{D_s^*} m_{D_s^*} \tilde{\eta}_\mu$$

$$\langle D_{s0}|J_0|0\rangle = f_{D_{s0}} m_{D_{s0}} \qquad (2.7)$$

so that F_μ can be written as

$$F_\mu = \frac{\langle D_{s0}|J_0|0\rangle\langle \gamma D_s^*|D_{s0}\rangle\langle 0|J_\mu^\dagger|D_s^*\rangle}{(m_{D_{s0}}^2-(p+q)^2)(m_{D_s^*}^2-p^2)} + \text{other resonances} + \text{continuum}\ , \qquad (2.8)$$

neglecting the widths of D_s^* and D_{s0}. The sum rule follows after a double Borel transformation in $-p^2$ and $-(p+q)^2$ of both the QCD and the hadronic representation of the correlation function, that involves two Borel parameters, M_1^2 and M_2^2. The transformation allows to suppress the contribution of the continuum of states and of higher resonances, to

suppress the higher twist terms in the QCD expression of the correlation function and to remove all terms that are either independent of one of the two variables $-p^2$ or $-(p+q)^2$ or depend on it only polynomially. The Borel parameters M_1^2 and M_2^2 are independent; we choose $M_1^2 = M_2^2$ since this allows, invoking global quark-hadron duality between the hadronic and the OPE expression of the correlation function above some threshold s_0, to subtract the continuum in the QCD side through the substitution $e^{-\frac{m_c^2}{M^2}} \longrightarrow e^{-\frac{m_c^2}{M^2}} - e^{-\frac{s_0}{M^2}}$ in the leading twist term [18]. The masses of the charmed mesons involved in the transitions are close to each other, therefore the choice of equal Borel parameters is reasonable. The final expression of the sum rule for d is:

$$
\begin{aligned}
d &= \frac{e^{\frac{m_{D_{s0}}^2+m_{D_s^*}^2}{2M^2}}}{m_{D_{s0}} f_{D_{s0}} m_{D_s^*} f_{D_s^*}} \Bigg\{ \int_{(m_c+m_s)^2}^{s_0} ds\, e^{-\frac{s}{M^2}} \rho^P(s) \\
&+ e_c\, e^{-\frac{m_c^2}{M^2}} \langle \bar{s}s \rangle \left(1 + \frac{m_s^2}{4M^2} + \frac{m_s^2 m_c^2}{2M^4}\right) + e_s \langle \bar{s}s \rangle (e^{-\frac{m_c^2}{M^2}} - e^{-\frac{s_0}{M^2}}) M^2 \chi \phi_\gamma(u_0) \\
&+ e_s \langle \bar{s}s \rangle e^{-\frac{m_c^2}{M^2}} \Bigg[-\frac{1}{4} \left(\mathbb{A}(u_0) - 8\bar{H}_\gamma(u_0)\right) \left(1 + \frac{m_c^2}{M^2}\right) \\
&+ \int_0^{1-u_0} dv \int_0^{\frac{u_0}{1-v}} d\alpha_g \mathcal{F}(u_0 - (1-v)\alpha_g, 1-u_0 - v\alpha_g, \alpha_g) \\
&+ \int_{1-u_0}^{1} dv \int_0^{\frac{1-u_0}{v}} d\alpha_g \mathcal{F}(u_0 - (1-v)\alpha_g, 1-u_0 - v\alpha_g, \alpha_g) \Bigg] \\
&- 2 e_s f_{3\gamma} m_c e^{-\frac{m_c^2}{M^2}} \Psi^v(u_0)) \Bigg\}
\end{aligned}
\tag{2.9}
$$

where $\mathcal{F} = \mathcal{S} - \tilde{\mathcal{S}} - T_1 + T_4 - T_3 + T_2 + 2v(-\mathcal{S} + T_3 - T_2)$, $\bar{H}_\gamma(u) = \int_0^u du' H_\gamma(u')$, $H_\gamma(u) = \int_0^u du' h_\gamma(u')$ and $\Psi^v(u) = \int_0^u du' \psi^v(u')$. All the distribution amplitudes are defined in the Appendix; $u_0 = \frac{M_1^2}{M_1^2+M_2^2} = \frac{1}{2}$.

The sum rule (2.9) involves the meson masses, for which we use the experimental data, and the leptonic constants $f_{D_s^*}$ and $f_{D_{s0}}$. For the former one, we put $f_{D_s^*} = f_{D_s}$ and use the central value of the experimental result $f_{D_s} = 266 \pm 32$ MeV [22]. As for $f_{D_{s0}}$, a sum rule obtained from the analysis of the correlation function

$$
\Pi(p^2) = i \int d^4x\, e^{ip\cdot x} \langle 0 | T[J_0(0) J_0^\dagger(x)] | 0 \rangle, \tag{2.10}
$$

$$f^2_{D_{s0}} = \frac{e^{\frac{m^2_{D_{s0}}}{M^2}}}{m^2_{D_{s0}}} \Bigg\{ \frac{3}{8\pi^2} \int_{(m_c+m_s)^2}^{s_0} ds\, \lambda^{\frac{1}{2}}(s, m_c^2, m_s^2) \left[1 - \frac{(m_c+m_s)^2}{s}\right] e^{-\frac{s}{M^2}}$$

$$+ \frac{e^{-\frac{m_c^2}{M^2}}}{2} \left[\langle \bar{s}s \rangle \left(2m_c - m_s - \frac{m_c^2 m_s}{M^2} + \frac{m_c^3 m_s^2}{M^4} \right) - \frac{\langle \bar{s}\sigma g_s G s\rangle}{2} \frac{m_c^3}{M^4} \right] \Bigg\} \qquad (2.11)$$

allows to obtain $f_{D_{s0}} = 225 \pm 25$ MeV, using the parameters in the Appendix.

From eq.(2.9) we can compute d varying the threshold s_0 and considering the range of the external variable M^2 where the result is independent on it (stability region). In this region a hierarchy in the terms with increasing twist is observed, so that we can presume that the neglect of higher-twist contributions induces a small error. On the other hand, the perturbative term, which depends on both the light and the heavy quark charges, represents a sizeable contribution to the sum rule.

In fig.3 we plot the curves corresponding to different values of s_0. Considering the range

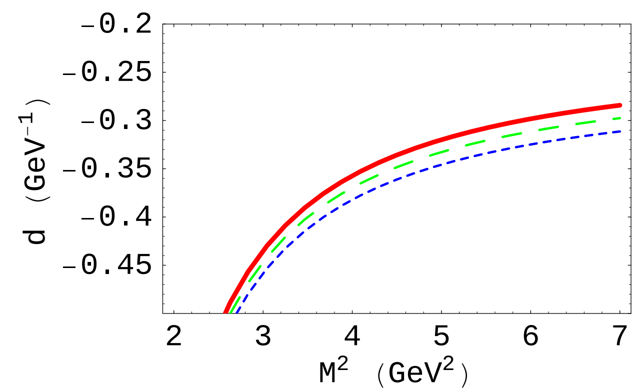

FIG. 3: The parameter d in the $D_{s0} \to D_s^* \gamma$ decay amplitude eq.(2.1) versus the Borel parameter M^2 . The curves correspond to the thresholds $s_0 = 2.45^2$ GeV^2 (continuous line), $s_0 = 2.5^2$ GeV^2 (long-dashed line) and $s_0 = 2.55^2$ GeV^2 (dashed line).

5 GeV$^2 \leq M^2 \leq 7$ GeV^2, where the best stability in M^2 is found, together with the variation of the threshold s_0, we get: -0.35 GeV$^{-1} \leq d \leq -0.28$ GeV^{-1}, corresponding to the radiative decay width

$$\Gamma(D_{s0} \to D_s^* \gamma) = (4\ -\ 6)\ \text{keV}. \qquad (2.12)$$

In (2.12) we have only considered the uncertainty in s_0 and M^2, and we have used the central values of the QCD parameters collected in the Appendix. Actually, such parameters represent another source of uncertainty. In particular, an important input parameter is the

magnetic susceptibility of the quark condensate, χ, for which we use the value determined in ref.[28]: $\chi = -(3.15 \pm 0.3)\,\mathrm{GeV}^{-2}$. A different value $\chi = -4.4\,\mathrm{GeV}^{-2}$, previously used in other sum rule analyses, would produce a 40% larger value of $|d|$.

The result (2.12) shows that the radiative decay occurs at a typical rate for this kind of transitions (a few keV's). However, the rate is larger by a factor of 4-5 than that obtained using VMD and the infinite heavy quark limit, and by a factor of 2-3 larger than the estimates based on the constituent quark model. It is interesting to investigate the reason of the numerical differences; aiming at that, we estimate d by light-cone QCD sum rules in the heavy quark limit, using an approach based on the heavy quark effective theory. We discuss such a calculation in the next Section.

III. $D_{sJ}(2317) \to D_s^* \gamma$ IN THE HEAVY QUARK LIMIT

In order to determine the hadronic parameter d in eq.(2.1) when $m_c \to \infty$, we consider two different correlation functions:

$$F^{(S)}_\mu(\omega, q\cdot v) = i \int d^4x\, e^{i(\omega v - q)\cdot x} \langle \gamma(q,\lambda)|T[\hat{J}^\dagger_\mu(x)\hat{J}_0(0)]|0\rangle \tag{3.1}$$

and

$$F^{(D)}_\mu(\omega, q\cdot v) = i \int d^4x\, e^{i(\omega v - q)\cdot x} \langle \gamma(q,\lambda)|T[\hat{J}^\dagger_\mu(x)\hat{J}_d(0)]|0\rangle \ . \tag{3.2}$$

The currents in (3.1) and (3.2) are effective currents constructed in terms of the strange quark field and of the effective field h_v of the heavy quark (in our case the charm quark) with four-velocity v. The effective field h_v is related to the heavy quark field Q in QCD through the relation $h_v = e^{im_Q v\cdot x}\dfrac{1+\not{v}}{2}Q$ (for a review see [23]). The current $\hat{J}_\mu = \bar{h}_v \gamma_\mu s$ has the quantum numbers of a vector meson. On the other hand, the currents $\hat{J}_0 = \bar{h}_v s$ and $\hat{J}_d = \bar{h}_v(-i)\gamma^\mu \vec{D}_{t\mu} s$ have both the quantum numbers of a scalar meson, since $D_{t\mu} \equiv g_{t\mu\alpha}D^\alpha \equiv (g_{\mu\alpha} - v_\mu v_\alpha)D^\alpha$, D being the covariant derivative. The latter current has been proposed as better suited for describing scalar heavy-light quark mesons in the heavy quark limit [24], therefore it is interesting to investigate how it behaves in sum rules for radiative decays.

The sum rules for d are obtained from (3.1) and (3.2) using the same procedure followed in Sec.II, namely considering the light-cone expansion and the hadronic representation of the correlation functions, making a double Borel transform in the variables ω and $\omega' = \omega - q\cdot v$

that involve two Borel parameters $E_{1,2}$, choosing $E_1 = E_2 = 2E$ and invoking quark-hadron duality above some threshold ν_0. From (3.1) we obtain [25]

$$
\begin{aligned}
d^{(S)} &= \frac{4}{\hat{F}\hat{F}^{+}} e^{\frac{\bar{\Lambda}+\bar{\Lambda}^{+}}{2E}} \Bigg\{ \frac{3 m_s e_s}{8\pi^2} \int_{m_s}^{\nu_0} d\nu \, e^{-\frac{\nu}{E}} \ln \left[\frac{\nu - (\nu^2 - m_s^2)^{\frac{1}{2}}}{\nu + (\nu^2 - m_s^2)^{\frac{1}{2}}} \right] \\
&+ e_s \frac{\langle \bar{s} s \rangle}{2} E \chi \phi_\gamma(u_0) \left(1 - e^{-\frac{\nu_0}{E}}\right) - e_s \frac{\langle \bar{s} s \rangle}{4E} \left(\frac{\mathbb{A}(u_0)}{8} - \bar{H}_\gamma(u_0) \right) - \frac{e_s f_{3\gamma}}{2} \Psi^v(u_0) \Bigg\} \; .
\end{aligned}
\tag{3.3}
$$

On the other hand, from the correlation function (3.2) we get:

$$
\begin{aligned}
d^{(D)} &= \frac{4}{\hat{F}\hat{F}_d^{+}} e^{\frac{\bar{\Lambda}+\bar{\Lambda}^{+}}{2E}} \Bigg\{ - \frac{3 m_s e_s}{8\pi^2} \int_{m_s}^{\nu_0} d\nu \, e^{-\frac{\nu}{E}} \ln \left[\frac{\nu - (\nu^2 - m_s^2)^{\frac{1}{2}}}{\nu + (\nu^2 - m_s^2)^{\frac{1}{2}}} \right] (m_s + \nu) \\
&+ E \frac{e_s f_{3\gamma}}{2} \left[\Psi^v(u_0) + \frac{1}{4} \psi^a(u_0) - u_0 \frac{\psi'^a(u_0)}{4} \right] \left(1 - e^{-\frac{\nu_0}{E}}\right) \\
&+ e_s \langle \bar{s} s \rangle \frac{1}{2} \left[- E^2 \left(\chi \phi_\gamma(u_0) + u_0 \chi \phi'_\gamma(u_0) \right) \right] \left(1 - e^{-\frac{\nu_0}{E}} (1 + \frac{\nu_0}{E}) \right) \\
&+ e_s \langle \bar{s} s \rangle \frac{1}{2} \left[\frac{1}{16} \left(\mathbb{A}(u_0) + u_0 \mathbb{A}'(u_0) \right) - \bar{H}_\gamma(u_0) \right] \Bigg\} \; .
\end{aligned}
\tag{3.4}
$$

Notice that in (3.3)-(3.4) only photon emission from the light quark contributes. In the heavy quark limit the current-vacuum matrix elements are defined as follows: $\langle 0|\hat{J}^\mu|D_s^*(v,\lambda)\rangle_H = \hat{F}\tilde{\eta}^\mu(\lambda)$, $\langle 0|\hat{J}_0|D_{s0}(v)\rangle_H = \hat{F}^+$, $\langle 0|\hat{J}_d|D_{s0}(v)\rangle_H = \hat{F}_d^+$ (the subscript H indicates that the states are normalized as used in HQET; $\hat{F}^{(+)}$ and $\hat{F}_d^+$ have dimension mass$^{3/2}$ and mass$^{5/2}$, respectively). Moreover, $\bar{\Lambda}$ and $\bar{\Lambda}^+$ are mass parameters defined as $\bar{\Lambda} = m_{D_s^*} - m_c$, $\bar{\Lambda}^+ = m_{D_{s0}} - m_c$ (in the heavy quark limit). We use the numerical values: $\hat{F} = 0.35$ GeV$^{\frac{3}{2}}$, $\hat{F}^+ = 0.45$ GeV$^{\frac{3}{2}}$, $\hat{F}_d^+ = 0.44$ GeV$^{\frac{5}{2}}$ and $\bar{\Lambda} = 0.5$ GeV, $\bar{\Lambda}^+ = 0.86$ GeV [23, 24, 26, 27]. In fig.4 (left) we depict the result corresponding to eq.(3.3). Considering the region where $d^{(S)}$ is independent of the Borel parameter E: $1.2\,\text{GeV} \leq E \leq 1.6\,\text{GeV}$, and the variation of the threshold ν_0, we obtain $-0.16\text{GeV}^{-1} \leq d^{(S)} \leq -0.13$ GeV^{-1}. Therefore, we obtain in the heavy quark limit a value compatible with the value obtained by VMD in the same

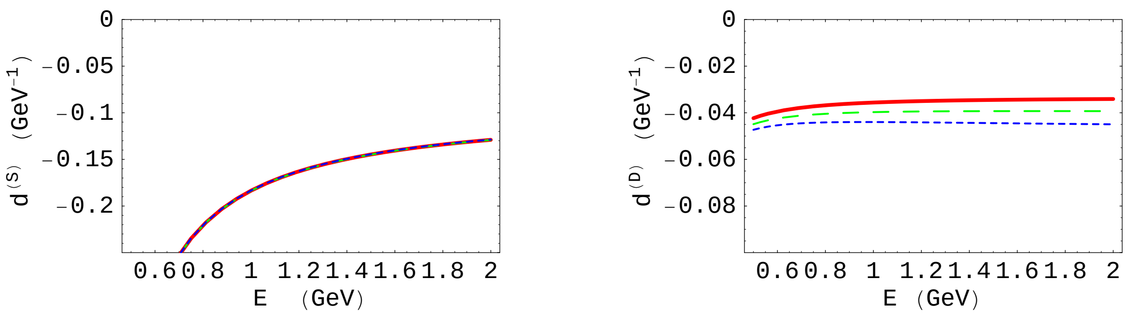

FIG. 4: The parameters $d^{(S)}$ obtained from eq.(3.3) (left) and $d^{(D)}$ from eq.(3.4) (right) versus the Borel parameter E. The continuous, long-dashed and dashed lines correspond to the thresholds $\nu_0 = 1.1, 1.2$ and 1.3 GeV, respectively.

limit: $d \simeq -0.15$ GeV^{-1}; finite quark mass effects are large, and enhance the $D_{s0} \to D_s^* \gamma$ amplitude by at least a factor of two.

From the second sum rule eq.(3.4), taking into account the dependence on the Borel parameter E for the continuum subtraction, we obtain a smaller result, see fig. 4 (right). This is due to a nearly complete cancellation between two different terms, the perturbative and the leading twist term, and therefore it critically depends on the input parameters of the QCD side of the sum rule, making the numerical result less reliable.

IV. $D_{sJ}(2460) \to D_s \gamma$

Coming back to the case of finite charm quark mass, let us consider three radiative decay modes of D'_{s1}, the transitions into a pseudoscalar D_s, a vector D_s^* and a scalar D_{s0} meson with the emission of a photon. The calculation of the decay amplitudes is analogous to the one carried out in Section II, therefore we present only the relevant formulae.

The decay amplitude of $D'_{s1} \to D_s \gamma$:

$$\langle \gamma(q, \lambda) D_s(p) | D'_{s1}(p + q, \lambda'') \rangle = e g_1 \left[(\varepsilon^* \cdot \eta)(p \cdot q) - (\varepsilon^* \cdot p)(\eta \cdot q) \right] \tag{4.1}$$

with $\eta(\lambda'')$ the D'_{s1} polarization vector, involves the hadronic parameter g_1 that can be computed considering the correlation function

$$T_\mu(p, q) = i \int d^4x \; e^{ip\cdot x} \langle \gamma(q, \lambda) | T[J_5^\dagger(x) J_\mu^A(0)] | 0 \rangle \; . \tag{4.2}$$

The quark currents are $J_5 = \bar{c} i \gamma_5 s$ and $J_\mu^A = \bar{c} \gamma_\mu \gamma_5 s$; T_μ can be expanded in Lorentz-invariant

structures:

$$T_\mu(p,q) = T\ \left[(\varepsilon^*\cdot p)q_\mu - (p\cdot q)\varepsilon^*_\mu\right] + ...\ . \tag{4.3}$$

The sum rule for g_1, obtained from the function T, reads:

$$\begin{aligned} g_1 &= \frac{e^{\frac{m^2_{D'_{s1}}+m^2_{D_s}}{2M^2}}(m_c+m_s)}{m_{D'_{s1}}f_{D'_{s1}}m^2_{D_s}f_{D_s}}\Bigg\{\int_{(m_c+m_s)^2}^{s_0} ds\ e^{-\frac{s}{M^2}}\rho^P(s) \\ &+\ e_c\, e^{-\frac{m_c^2}{M^2}}\langle\bar{s}s\rangle\left[1-\frac{m_c m_s}{M^2}+\frac{m_s^2}{2M^2}\left(1+\frac{m_c^2}{M^2}\right)\right] \\ &-\ e_s\langle\bar{s}s\rangle(e^{-\frac{m_c^2}{M^2}}-e^{-\frac{s_0}{M^2}})M^2\chi\phi_\gamma(u_0) \\ &-\ e_s\langle\bar{s}s\rangle e^{-\frac{m_c^2}{M^2}}\Bigg[-\frac{1}{4}\left(\mathbb{A}(u_0)-8\bar{H}_\gamma(u_0)\right)\left(1+\frac{m_c^2}{M^2}\right) \\ &-\ \int_0^{1-u_0}dv\int_0^{\frac{u_0}{1-v}}d\alpha_g\mathcal{F}_1(u_0-(1-v)\alpha_g, 1-u_0-v\alpha_g,\alpha_g) \\ &-\ \int_{1-u_0}^{1}dv\int_0^{\frac{1-u_0}{v}}d\alpha_g\mathcal{F}_1(u_0-(1-v)\alpha_g, 1-u_0-v\alpha_g,\alpha_g)\Bigg] \\ &+\ 2e_s f_{3\gamma}m_c e^{-\frac{m_c^2}{M^2}}\Psi^v(u_0)\Bigg\}\ , \end{aligned} \tag{4.4}$$

where $\mathcal{F}_1 = \mathcal{S}+\tilde{\mathcal{S}}-T_1-T_2+T_3+T_4+2v(-\mathcal{S}-T_3+T_2)$ and the spectral function ρ^P is:

$$\begin{aligned} \rho^P(s) &= -\frac{3e_s}{8\pi^2}\Bigg\{2m_s\ln\left(\frac{s-m_c^2+m_s^2-\lambda^{\frac{1}{2}}(s,m_c^2,m_s^2)}{s-m_c^2+m_s^2+\lambda^{\frac{1}{2}}(s,m_c^2,m_s^2)}\right) \\ &\quad+\ (m_c-m_s)\frac{(m_c^2-m_s^2-s)}{s^2}\lambda^{\frac{1}{2}}(s,m_c^2,m_s^2)\Bigg\}\ -\ (s\leftrightarrow c)\ . \end{aligned} \tag{4.5}$$

Eq.(4.4) involves parameters already used in previous Sections and the photon DA collected in the Appendix; it also involves the leptonic constant $f_{D'_{s1}}$ defined by the matrix element

$$\langle 0|J^A_\mu|D'_{s1}\rangle = f_{D'_{s1}}m_{D'_{s1}}\eta_\mu\ , \tag{4.6}$$

which can be obtained, starting from the two-point correlation function

$$\Pi_{\mu\nu}(p^2) = i\int d^4x\ e^{ip\cdot x}\langle 0|T[J^A_\mu(0)J^{A\dagger}_\nu(x)]|0\rangle\ , \tag{4.7}$$

from the sum rule:

$$f^2_{D'_{s1}} = \frac{e^{\frac{m^2_{D'_{s1}}}{M^2}}}{m^2_{D'_{s1}}}\Bigg\{\frac{1}{8\pi^2}\int_{(m_c+m_s)^2}^{s_0} ds\, \lambda^{\frac{1}{2}}(s,m_c^2,m_s^2)\left[2-\frac{m_c^2+m_s^2+6m_cm_s}{s}-\frac{(m_c^2-m_s^2)^2}{s^2}\right]e^{-\frac{s}{M^2}}$$

$$+\; e^{-\frac{m_c^2}{M^2}}\left[\langle\bar{s}s\rangle\left(m_c-\frac{m_c^2m_s}{2M^2}+\frac{m_c^3m_s^2}{2M^4}\right)-\frac{\langle\bar{s}\sigma g_sGs\rangle}{4}\frac{m_c^3}{M^4}\right]\Bigg\}\;. \tag{4.8}$$

We get $f_{D'_{s1}} \simeq f_{D_{s0}}$.

The calculation of g_1 produces the curves depicted in fig.5. Considering the range

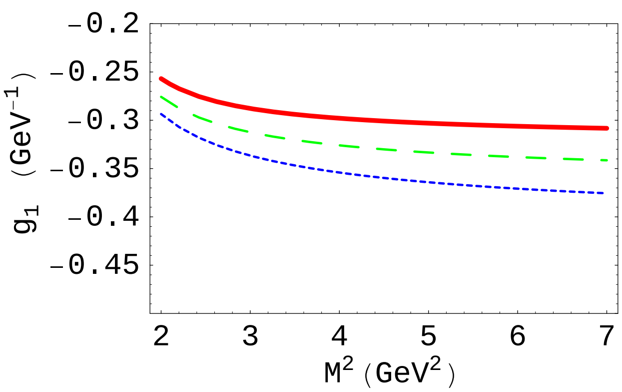

FIG. 5: The parameter g_1 in the $D'_{s1}\to D_s^*\gamma$ decay amplitude, eq.(4.1), as a function of the Borel parameter M^2. The curves refer to the threshold $s_0 = 2.5^2$ GeV^2 (continuous), $s_0 = 2.55^2$ GeV^2 (long-dashed) and $s_0 = 2.6^2$ GeV^2 (dashed line).

3 GeV^2 $\le$ M^2 $\le$ 6 GeV^2, together with the variation of the threshold s_0, we obtain: -0.37 GeV^{-1} $\le g_1 \le -0.29$ GeV^{-1}, and therefore

$$\Gamma(D'_{s1}\to D_s\gamma) = (19\,-\,29)\ \text{keV}. \tag{4.9}$$

As for $D_{s0}\to D_s^*\gamma$, the result of light-cone sum rules for the width of $D'_{s1}\to D_s\gamma$ is larger than previous estimates. We shall discuss this point later on.

V. $D_{sJ}(2460)\to D_s^*\gamma$

The calculation of the dimensionless hadronic parameter g_2 appearing in the $D'_{s1}\to D_s^*\gamma$ transition amplitude:

$$\langle\gamma(q,\lambda)D_s^*(p,\lambda')|D'_{s1}(p+q,\lambda'')\rangle = i\,e\,g_2\,\varepsilon_{\alpha\beta\sigma\tau}\eta^\alpha\tilde{\eta}^{*\beta}\varepsilon^{*\sigma}q^\tau\;\;, \tag{5.1}$$

with $\tilde{\eta}(\lambda')$ and $\eta(\lambda'')$ the polarization vectors of D_s^* and D'_{s1}, is based on the analysis of the correlation function

$$T_{\mu\nu}(p,q) = i\int d^4x\, e^{ip\cdot x}\langle\gamma(q,\lambda)|T[J_\mu^\dagger(x)J_\nu^A(0)]|0\rangle \tag{5.2}$$

expanded in Lorentz invariant structures

$$\begin{aligned} T_{\mu\nu}(p,q) &= T_A\,\varepsilon_{\mu\nu\sigma\tau}\varepsilon^{*\sigma}q^\tau + T_B\,p_\mu\varepsilon_{\nu\beta\sigma\tau}p^\beta\varepsilon^{*\sigma}q^\tau \\ &+ T_C\,(p+q)_\nu\varepsilon_{\alpha\mu\sigma\tau}p^\alpha\varepsilon^{*\sigma}q^\tau + \dots\ . \end{aligned} \tag{5.3}$$

A sum rule for g_2 is obtained from T_A:

$$\begin{aligned} g_2 \;=\; & \frac{e^{\frac{m^2_{D'_{s1}}+m^2_{D_s^*}}{2M^2}}}{m_{D'_{s1}}f_{D'_{s1}}m_{D_s^*}f_{D_s^*}}\Bigg\{\int_{(m_c+m_s)^2}^{s_0} ds\, e^{-\frac{s}{M^2}}\rho^P(s) + e_c m_c\, e^{-\frac{m_c^2}{M^2}}\langle\bar{s}s\rangle\left[1-\frac{m_s^2}{M^2}\left(1-\frac{m_c^2}{M^2}\right)\right] \\ &+\; e_s m_c\langle\bar{s}s\rangle(e^{-\frac{m_c^2}{M^2}}-e^{-\frac{s_0}{M^2}})M^2\chi\phi_\gamma(u_0) \\ &+\; e_s m_c\langle\bar{s}s\rangle e^{-\frac{m_c^2}{M^2}}\left[-\frac{1}{4}\frac{m_c^2}{M^2}\mathbb{A}(u_0) - H_\gamma(u_0)(1-u_0) - \bar{H}_\gamma(u_0)\left(1-\frac{2m_c^2}{M^2}\right)\right] \\ &+\; e_s f_{3\gamma}M^2(e^{-\frac{m_c^2}{M^2}}-e^{-\frac{s_0}{M^2}})\Bigg[\frac{1}{4}(1-u_0)\psi'^a(u_0) - \frac{1}{4}\psi^a(u_0) - \Psi^v(u_0)\left(1+\frac{2m_c^2}{M^2}\right) \\ &+\; (1-u_0)\psi^v(u_0)\Bigg] \\ &+\; m_c e_s\langle\bar{s}s\rangle e^{-\frac{m_c^2}{M^2}}\Bigg[\int_0^{1-u_0}dv\int_0^{\frac{u_0}{1-v}}d\alpha_g\mathcal{F}_2(u_0-(1-v)\alpha_g, 1-u_0-v\alpha_g,\alpha_g) \\ &+\; \int_{1-u_0}^1 dv\int_0^{\frac{1-u_0}{v}}d\alpha_g\mathcal{F}_2(u_0-(1-v)\alpha_g,1-u_0-v\alpha_g,\alpha_g)\Bigg] \\ &-\; e_s f_{3\gamma}M^2(e^{-\frac{m_c^2}{M^2}}-e^{-\frac{s_0}{M^2}})\Bigg[\int_0^{u_0}d\alpha_{\bar{q}}\int_{u_0-\alpha_{\bar{q}}}^{1-\alpha_{\bar{q}}}\frac{d\alpha_g}{\alpha_g^2}\,\mathcal{F}_3(1-\alpha_{\bar{q}}-\alpha_g,\alpha_{\bar{q}},\alpha_g) \\ &-\; \int_0^{u_0}d\alpha_{\bar{q}}\frac{1}{u_0-\alpha_{\bar{q}}}\;\mathcal{F}_3(1-u_0,\alpha_{\bar{q}},u_0-\alpha_{\bar{q}})\Bigg]\Bigg\}\ , \end{aligned} \tag{5.4}$$

with $\mathcal{F}_2 = \mathcal{S} + \tilde{\mathcal{S}} + T_1 - T_2 - T_3 + T_4$ and $\mathcal{F}_3 = \mathcal{A} + \mathcal{V}$. The perturbative spectral function ρ^P reads:

$$\rho^P(s) = \frac{3e_s}{4\pi^2} m_s m_c \ln\left(\frac{s - m_c^2 + m_s^2 - \lambda^{\frac{1}{2}}(s, m_c^2, m_s^2)}{s - m_c^2 + m_s^2 + \lambda^{\frac{1}{2}}(s, m_c^2, m_s^2)}\right) + (s \leftrightarrow c) \ . \tag{5.5}$$

The result is reported in fig.6. Considering the range $4 \text{ GeV}^2 \leq M^2 \leq 6 \text{ GeV}^2$ and the

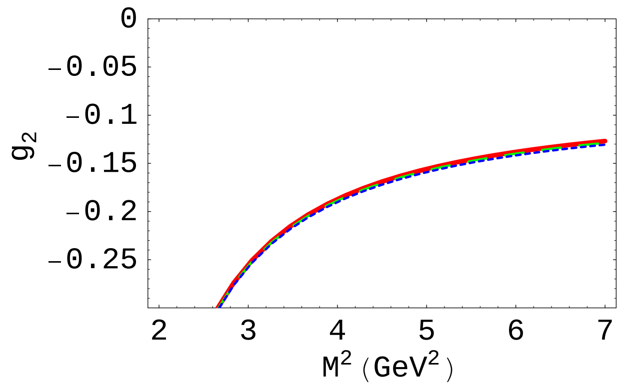

FIG. 6: The parameter g_2 in the $D'_{s1} \to D_s^* \gamma$ amplitude eq.(5.1) versus the Borel parameter M^2. The continuous, long-dashed and dashed lines refer to $s_0 = 2.5^2$ GeV^2, $s_0 = 2.55^2$ GeV^2 and $s_0 = 2.6^2$ GeV^2, respectively.

variation of the threshold s_0, we get $-0.18 \ \leq g_2 \leq -0.13$, i.e.

$$\Gamma(D'_{s1} \to D_s^* \gamma) = (0.6 - 1.1) \text{ keV}. \tag{5.6}$$

The small value of g_2 is due to large cancellations between the various contributions to the sum rule (5.4): perturbative, twist two and higher twist contributions, as shown in fig.7. In particular, the contribution proportional to $f_{3\gamma}$ turns out to be 50% of the contribution proportional to the magnetic susceptibility of the quark condensate. In the cancellation the detailed shapes of the distribution amplitudes and the numerical values of the parameters are of critical importance; this sensitivity induces us to consider the result for g_2 as less accurate than the results for the other channels.

VI. $D_{sJ}(2460) \to D^*_{sJ}(2317)\gamma$

The last radiative decay mode we consider for $D_{sJ}(2460)$ is the M1 transition $D'_{s1} \to D_{s0}\gamma$ which is governed by the amplitude

$$\langle \gamma(q, \lambda) D_{s0}(p) | D'_{s1}(p + q, \lambda'') \rangle = i \, e \, g_3 \, \varepsilon_{\alpha\beta\sigma\tau} \varepsilon^{*\alpha} \eta^\beta p^\sigma q^\tau \ . \tag{6.1}$$

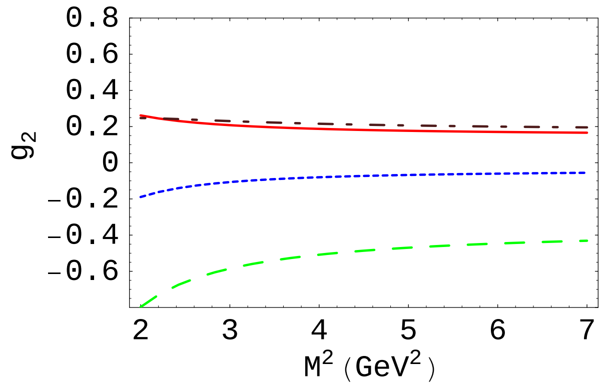

FIG. 7: Contributions to the sum rule (5.4) for g_2. The continuous line corresponds to the perturbative contribution in fig.2 (a,b), the long-dashed line to the term proportional to the magnetic susceptibility of the quark condensate χ, the long-short dashed line to the contribution proportional to $f_{3\gamma}$ and the dashed line to the contribution corresponding to fig.2 (c). The threshold is fixed to $s_0 = 2.55^2$ GeV^2 .

The parameter g_3 can be evaluated starting from the correlation function

$$W_\mu(p,q) = i \int d^4x \; e^{ip\cdot x} \langle \gamma(q,\lambda)|T[J_0^\dagger(x) J_\mu^A(0)]|0\rangle \tag{6.2}$$

written as

$$W_\mu = i \; \varepsilon_{\mu\alpha\sigma\tau}\varepsilon^{*\alpha} p^\sigma q^\tau \;\; W_0 \;\; . \tag{6.3}$$

We work out the sum rule for g_3:

$$\begin{aligned}
g_3 \; = \; & \frac{e^{\frac{m^2_{D'_{s1}}+m^2_{D_{s0}}}{2M^2}}}{m_{D_{s0}} f_{D_{s0}} m_{D'_{s1}} f_{D'_{s1}}} \Bigg\{ \int_{(m_c+m_s)^2}^{s_0} ds \; e^{-\frac{s}{M^2}} \rho^P(s) + e_c \; e^{-\frac{m_c^2}{M^2}} \langle \bar s s\rangle \left(1 + \frac{m_s m_c}{2M^2} + \frac{m_s^2 m_c^2}{8M^4}\right) \\
& + \; e_s \langle \bar s s\rangle (e^{-\frac{m_c^2}{M^2}} - e^{-\frac{s_0}{M^2}}) M^2 \chi \phi_\gamma(u_0) \\
& + \; e^{-\frac{m_c^2}{M^2}} e_s \langle \bar s s\rangle [-\frac{1}{4}\mathbb{A}(u_0)(1+\frac{m_c^2}{M^2})] - \frac{m_c}{2} e_s f_{3\gamma} \psi^a(u_0) e^{-\frac{m_c^2}{M^2}} \\
& + \; e^{-\frac{m_c^2}{M^2}} e_s \langle \bar s s\rangle \Big[\int_0^{1-u_0} dv \int_0^{\frac{u_0}{1-v}} d\alpha_g \mathcal{F}_4(u_0-(1-v)\alpha_g, 1-u_0-v\alpha_g, \alpha_g) \\
& + \; \int_{1-u_0}^1 dv \int_0^{\frac{1-u_0}{v}} d\alpha_g \mathcal{F}_4(u_0-(1-v)\alpha_g, 1-u_0-v\alpha_g, \alpha_g) \Big] \Bigg\}
\end{aligned} \tag{6.4}$$

with $\mathcal{F}_4 = \mathcal{S} + \tilde{\mathcal{S}} + T_1 + T_4 - T_2 - T_3 + 2v(-\tilde{\mathcal{S}} + T_3 - T_4)$ and

$$\rho^P(s) = \frac{3e_s}{4\pi^2}\left\{\frac{(m_c+m_s)}{s}\lambda^{\frac{1}{2}}(s,m_c^2,m_s^2) + m_s \ln\left(\frac{s-m_c^2+m_s^2-\lambda^{\frac{1}{2}}(s,m_c^2,m_s^2)}{s-m_c^2+m_s^2+\lambda^{\frac{1}{2}}(s,m_c^2,m_s^2)}\right)\right\}$$
$$- \ (s \leftrightarrow c) \ . \tag{6.5}$$

Considering the range $4 \text{ GeV}^2 \leq M^2 \leq 6 \text{ GeV}^2$ and varying the threshold s_0 we get:

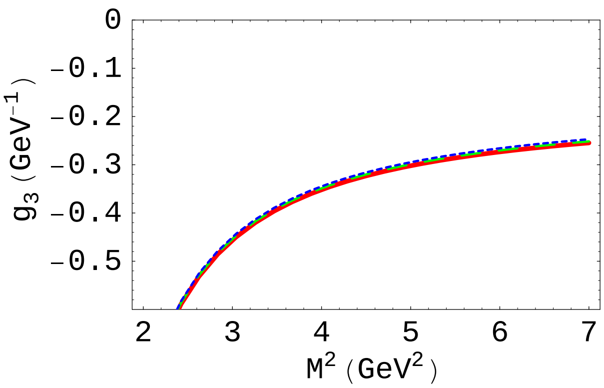

FIG. 8: The parameter g_3 in the $D'_{s1} \to D_{s0}\gamma$ amplitude, eq.(6.1), versus the Borel parameter M^2. The curves correspond to the same thresholds as in fig. 5 and 6.

$-0.35 \text{ GeV}^{-1} \leq g_3 \leq -0.27 \text{ GeV}^{-1}$, corresponding to

$$\Gamma(D'_{s1} \to D_{s0}\gamma) = (0.5 - 0.8) \text{ keV}. \tag{6.6}$$

VII. DISCUSSION AND CONCLUSIONS

As seen in the previous Sections, the radiative decay amplitudes of the charmed mesons considered here, when evaluated by light-cone QCD sum rules, are determined by two main contributions, the perturbative photon emission from the heavy and light quarks, and the contribution of the photon emission from the soft light quark. Other terms represent small corrections. In general, these two terms have different signs, and produce large cancellations; this allows to understand the role of QCD parameters like the magnetic susceptibility χ. The delicate balancing of the two contributions determines the difference between the radiative widths of charged and neutral mesons.

In Table I we collect the LCSR results together with the results of other methods [10, 11, 15, 16]. With the exception of $D_{sJ}(2460) \to D_s^*\gamma$, the rates of all the modes are larger

TABLE I: Radiative decay widths (in keV) of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ obtained by light-cone sum rules (LCSR). Vector Meson Dominance (VMD) and constituent quark model (QM) results are also reported.

Initial state	Final state	LCSR	VMD [10, 16]	QM [15]	QM [11]
$D^*_{sJ}(2317)$	$D^*_s\gamma$	4-6	0.85	1.9	1.74
$D_{sJ}(2460)$	$D_s\gamma$	19-29	3.3	6.2	5.08
	$D^*_s\gamma$	0.6-1.1	1.5	5.5	4.66
	$D^*_{sJ}(2317)\gamma$	0.5-0.8	—	0.012	2.74

than obtained by other approaches. In particular, $\Gamma(D_{sJ}(2460) \to D_s\gamma)$ turns out to be considerably wider; notice that $D_{sJ}(2460) \to D_s\gamma$ is the only radiative mode observed so far, as shown in Table II. The peculiarity in $D_{sJ}(2460) \to D_s\gamma$ is that the perturbative contribution to the sum rule is the largest term, while in the other cases the leading twist term is the largest one in the theoretical side of the sum rules.

TABLE II: Measurements and 90% CL upper limits of ratios of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ decay widths.

	Belle	BaBar	CLEO [2]
$\frac{\Gamma\left(D^*_{sJ}(2317)\to D^*_s\gamma\right)}{\Gamma\left(D^*_{sJ}(2317)\to D_s\pi^0\right)}$	< 0.18 [3]	—	< 0.059
$\frac{\Gamma(D_{sJ}(2460)\to D_s\gamma)}{\Gamma(D_{sJ}(2460)\to D^*_s\pi^0)}$	$0.55 \pm 0.13 \pm 0.08$ [3]	$0.375 \pm 0.054 \pm 0.057$ [9]	< 0.49
	$0.38 \pm 0.11 \pm 0.04$ [4]	$0.274 \pm 0.045 \pm 0.020$ [7]	
$\frac{\Gamma(D_{sJ}(2460)\to D^*_s\gamma)}{\Gamma(D_{sJ}(2460)\to D^*_s\pi^0)}$	< 0.31 [3]	—	< 0.16
$\frac{\Gamma\left(D_{sJ}(2460)\to D^*_{sJ}(2317)\gamma\right)}{\Gamma(D_{sJ}(2460)\to D^*_s\pi^0)}$	—	< 0.23 [9]	< 0.58

In order to quantitatively understand the data in Table II one should precisely know the widths of the isospin violating transitions $D_{s0} \to D_s\pi^0$ and $D'_{s1} \to D^*_s\pi^0$. In the description of these transitions based on the mechanism of $\eta - \pi^0$ mixing [13, 15, 16, 30] one should accurately determine the strong couplings $D_{s0}D_s\eta$ and $D'_{s0}D^*_s\eta$ for finite heavy quark mass. Considering the results in Tables I,II, these couplings should be larger than obtained in the heavy quark and SU(3) limit, an issue which deserves further investigation.

The obtained dominance of $D'_{s1} \to D_s\gamma$ with respect to other modes, in agreement with observation, induces us to consider our results as consistent with the interpretation of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ as ordinary $\bar{c}s$ mesons. The $D_{sJ}(2460) \to D^*_{sJ}(2317)\gamma$ decay turns out to be suppressed; in the molecular interpretation it would be somehow enhanced. The observation of all the radiative decay modes with the predicted rates would of course reinforce our statement; in the meanwhile, we can reasonably conclude that invoking non-standard interpretations is not necessary.

Acnowledgments. We thank T. Aliev for discussions and for collaboration in the early stage of this work. We acknowledge partial support from the EC Contract No. HPRN-CT-2002-00311 (EURIDICE).

APPENDIX A: PHOTON DISTRIBUTION AMPLITUDES

For completeness, we collect in this Appendix the light-cone expansions of the photon matrix elements relevant for the calculation of the radiative decays of D_{s0} and D'_{s1}. We also collect the expressions of the photon distribution amplitudes and the numerical values of the related parameters, as reported in [28]. In all the expressions $\varepsilon(\lambda)$ is the photon polarization vector and $\tilde{\varepsilon}_\mu = \varepsilon^*_\mu - q_\mu \dfrac{\varepsilon^* \cdot x}{q \cdot x}$, $\tilde{g}_{\mu\nu} = g_{\mu\nu} - \dfrac{1}{q \cdot x}(q_\mu x_\nu + q_\nu x_\mu)$; the variable $\bar{u}$ is defined as $\bar{u} = 1 - u$; $\tilde{G}_{\mu\nu}$ is the dual field $\tilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$. We neglect quark mass corrections, that have not been worked out for all matrix elements.

$$
\begin{aligned}
\langle \gamma(q,\lambda)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle &= -iee_q\langle\bar{q}q\rangle(\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu)\int_0^1 du\, e^{i\bar{u}q\cdot x}\left(\chi\phi_\gamma(u) + \frac{x^2}{16}\mathbb{A}(u)\right) \\
&- iee_q\frac{\langle\bar{q}q\rangle}{2qx}(x_\nu\tilde{\varepsilon}_\mu - x_\mu\tilde{\varepsilon}_\nu)\int_0^1 du\, e^{i\bar{u}q\cdot x}h_\gamma(u) \\
\langle \gamma(q,\lambda)|\bar{q}(x)\gamma_\mu q(0)|0\rangle &= ee_q f_{3\gamma}\tilde{\varepsilon}_\mu\int_0^1 du\, e^{i\bar{u}q\cdot x}\psi^v(u) \\
\langle \gamma(q,\lambda)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle &= -\frac{1}{4}ee_q f_{3\gamma}\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}q^\alpha x^\beta\int_0^1 du\, e^{i\bar{u}q\cdot x}\psi^a(u) \\
\langle \gamma(q,\lambda)|\bar{q}(x)g_s G_{\mu\nu}(vx)q(0)|0\rangle &= -iee_q\langle\bar{q}q\rangle(\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu)\int \mathcal{D}\alpha_i\, e^{i(\alpha_{\bar{q}}+v\alpha_g)q\cdot x}\mathcal{S}(\alpha_i) \\
\langle \gamma(q,\lambda)|\bar{q}(x)g_s G_{\mu\nu}(vx)i\gamma_5 q(0)|0\rangle &= -iee_q\langle\bar{q}q\rangle(\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu)\int \mathcal{D}\alpha_i\, e^{i(\alpha_{\bar{q}}+v\alpha_g)q\cdot x}\tilde{\mathcal{S}}(\alpha_i) \\
\langle \gamma(q,\lambda)|\bar{q}(x)g_s\tilde{G}_{\mu\nu}(vx)\gamma_\alpha\gamma_5 q(0)|0\rangle &= ee_q f_{3\gamma}q_\alpha(\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu)\int \mathcal{D}\alpha_i\, e^{i(\alpha_{\bar{q}}+v\alpha_g)q\cdot x}\mathcal{A}(\alpha_i) \\
\langle \gamma(q,\lambda)|\bar{q}(x)g_s G_{\mu\nu}(vx)i\gamma_\alpha q(0)|0\rangle &= ee_q f_{3\gamma}q_\alpha(\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu)\int \mathcal{D}\alpha_i\, e^{i(\alpha_{\bar{q}}+v\alpha_g)q\cdot x}\mathcal{V}(\alpha_i)
\end{aligned}
$$

$$
\begin{aligned}
\langle \gamma(q,\lambda)|\bar q(x)\sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0)|0\rangle \;=\;& \\
ee_q\langle\bar q q\rangle \Bigg\{ \Bigg[\tilde\varepsilon_\mu \tilde g_{\alpha\nu} q_\beta - \tilde\varepsilon_\mu \tilde g_{\beta\nu} q_\alpha \;-\; (\mu\leftrightarrow\nu)\Bigg] & \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+v\alpha_g) q\cdot x} \mathcal{T}_1(\alpha_i) \\
+\Bigg[\tilde\varepsilon_\alpha \tilde g_{\mu\beta} q_\nu - \tilde\varepsilon_\alpha \tilde g_{\nu\beta} q_\mu \;-\; (\alpha\leftrightarrow\beta)\Bigg] & \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+v\alpha_g) q\cdot x} \mathcal{T}_2(\alpha_i) \\
+\; \frac{(q_\mu x_\nu - q_\nu x_\mu)(\varepsilon^*_\alpha q_\beta - \varepsilon^*_\beta q_\alpha)}{q\cdot x} & \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+v\alpha_g) q\cdot x} \mathcal{T}_3(\alpha_i) \\
+\; \frac{(q_\alpha x_\beta - q_\beta x_\alpha)(\varepsilon^*_\mu q_\nu - \varepsilon^*_\nu q_\mu)}{q\cdot x} & \int \mathcal{D}\alpha_i e^{i(\alpha_{\bar q}+v\alpha_g) q\cdot x} \mathcal{T}_4(\alpha_i) \Bigg\}
\end{aligned}
\tag{A1}
$$

$\alpha_i = \{\alpha_q, \alpha_{\bar q}, \alpha_g\}$ and $\displaystyle\int \mathcal{D}(\alpha_i) \equiv \int_0^1 d\alpha_q \int_0^1 d\alpha_{\bar q} \int_0^1 d\alpha_g \delta(1-\alpha_q-\alpha_{\bar q}-\alpha_g)$. The photon distribution amplitudes (DA) have the following expressions:

$$
\begin{aligned}
\phi_\gamma(u) &= 6u\bar u\left(1+\varphi_2 C_2^{\frac{3}{2}}(2u-1)\right) \\
\mathbb{A}(u) &= 40u^2\bar u^2(3k-k^+ +1) + 8(\zeta_2^+ - 3\zeta_2)\big[u\bar u(2+13u\bar u) \\
&+ 2u^3(10-15u+6u^2)\ln u + 2\bar u^3(10-15\bar u+6\bar u^2)\ln\bar u\big] \\
h_\gamma(u) &= -10(1+2k^+)C_2^{\frac{1}{2}}(2u-1) \\
\psi^v(u) &= 5\left(3(2u-1)^2-1\right) + \frac{3}{64}\left(15\omega_\gamma^V - 5\omega_\gamma^A\right)\left(3-30(2u-1)^2+35(2u-1)^4\right) \\
\psi^a(u) &= \left(1-(2u-1)^2\right)\left(5(2u-1)^2-1\right)\frac{5}{2}\left(1+\frac{9}{16}\omega_\gamma^V - \frac{3}{16}\omega_\gamma^A\right) \\
\mathcal{V}(\alpha_q,\alpha_{\bar q},\alpha_g) &= 540\omega_\gamma^V(\alpha_q-\alpha_{\bar q})\alpha_q\alpha_{\bar q}\alpha_g^2 \\
\mathcal{A}(\alpha_q,\alpha_{\bar q},\alpha_g) &= 360\alpha_q\alpha_{\bar q}\alpha_g^2\left[1+\omega_\gamma^A\frac{1}{2}(7\alpha_g-3)\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}(\alpha_q,\alpha_{\bar q},\alpha_g) &= 30\alpha_g^2\Big[(k+k^+)(1-\alpha_g)+(\zeta_1+\zeta_1^+)(1-\alpha_g)(1-2\alpha_g)\\
&+ \zeta_2[3(\alpha_{\bar q}-\alpha_q)^2-\alpha_g(1-\alpha_g)]\Big]\\
\tilde{\mathcal{S}}(\alpha_q,\alpha_{\bar q},\alpha_g) &= -30\alpha_g^2\Big[(k-k^+)(1-\alpha_g)+(\zeta_1-\zeta_1^+)(1-\alpha_g)(1-2\alpha_g)\\
&+ \zeta_2[3(\alpha_{\bar q}-\alpha_q)^2-\alpha_g(1-\alpha_g)]\Big]\\
\mathcal{T}_1(\alpha_q,\alpha_{\bar q},\alpha_g) &= -120(3\zeta_2+\zeta_2^+)(\alpha_{\bar q}-\alpha_q)\alpha_q\alpha_{\bar q}\alpha_g\\
\mathcal{T}_2(\alpha_q,\alpha_{\bar q},\alpha_g) &= 30\alpha_g^2(\alpha_{\bar q}-\alpha_q)\left[(k-k^+)+(\zeta_1-\zeta_1^+)(1-2\alpha_g)+\zeta_2(3-4\alpha_g)\right]\\
\mathcal{T}_3(\alpha_q,\alpha_{\bar q},\alpha_g) &= -120(3\zeta_2-\zeta_2^+)(\alpha_{\bar q}-\alpha_q)\alpha_q\alpha_{\bar q}\alpha_g\\
\mathcal{T}_4(\alpha_q,\alpha_{\bar q},\alpha_g) &= 30\alpha_g^2(\alpha_{\bar q}-\alpha_q)\left[(k+k^+)+(\zeta_1+\zeta_1^+)(1-2\alpha_g)+\zeta_2(3-4\alpha_g)\right].
\end{aligned}
\tag{A2}
$$

The parameters in the distribution amplitudes are: $f_{3\gamma} = -(0.0039 \pm 0.0020)\,\mathrm{GeV}^2$, $\omega_\gamma^V = 3.8 \pm 1.8$, $\omega_\gamma^A = -2.1 \pm 1.0$ [28]; $k = 0.2$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\varphi_2 = k^+ = \zeta_1^+ = \zeta_2^+ = 0$ (at the renormalization scale $\mu = 1$ GeV) [19]. The other parameters in the QCD sides of the sum rules, at the same renormalization scale, are: $m_c = 1.35$ GeV, $m_s = 0.125$ GeV [29], $\langle\bar s s\rangle = 0.8\langle\bar q q\rangle$ ($q = u, d$), $\langle\bar q q\rangle = (-0.245\ \mathrm{GeV})^3$ and $\langle\bar q g\sigma G q\rangle = m_0^2\langle\bar q q\rangle$ with $m_0^2 = 0.8$ GeV^2. Finally, for the magnetic susceptibility of the quark condensate χ we use the value $\chi = -(3.15 \pm 0.3)$ GeV^{-2} obtained in [28].

[1] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **90**, 242001 (2003).

[2] D. Besson *et al.* [CLEO Collaboration], Phys. Rev. **D68**, 032002 (2003).

[3] Y. Mikami *et al.* [Belle Collaboration], Phys. Rev. Lett. **92**, 012002 (2004).

[4] P. Krokovny *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262002 (2003).

[5] A. Drutskoy *et al.* [Belle Collaboration], Phys. Rev. Lett. **96**, 061802 (2005).

[6] E. W. Vaandering [FOCUS Collaboration], arXiv:hep-ex/0406044.

[7] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **93**, 181801 (2004).

[8] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. **D69**, 031101 (2004).

[9] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0408067.

[10] P. Colangelo, F. De Fazio and R. Ferrandes, Mod. Phys. Lett. A **19**, 2083 (2004).

[11] W. A. Bardeen, E. J. Eichten and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).

[12] M. A. Nowak, M. Rho and I. Zahed, Acta Phys. Polon. B **35**, 2377 (2004).

[13] T. Barnes, F. E. Close and H. J. Lipkin, Phys. Rev. D **68**, 054006 (2003).

[14] E. van Beveren and G. Rupp, Phys. Rev. Lett. **91**, 012003 (2003).

[15] S. Godfrey, Phys. Lett. B **568**, 254 (2003).

[16] P. Colangelo and F. De Fazio, Phys. Lett. B **570**, 180 (2003).

[17] N. S. Craigie and J. Stern, Nucl. Phys. B **216**, 209 (1983); V. M. Braun and I. E. Filyanov, Z. Phys. C **44**, 157 (1989); V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. B **345** (1990) 137.

[18] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Ruckl, Phys. Rev. D **51**, 6177 (1995).

[19] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. B **312**, 509 (1989).

[20] A. Ali, V. M. Braun and H. Simma, Z. Phys. C **63**, 437 (1994); A. Khodjamirian, G. Stoll and D. Wyler, Phys. Lett. B **358**, 129 (1995); T. M. Aliev, D. A. Demir, E. Iltan and N. K. Pak, Phys. Rev. D **54**, 857 (1996).

[21] P. Colangelo and A. Khodjamirian, in 'At the Frontier of Particle Physics/Handbook of QCD', ed. by M. Shifman (World Scientific, Singapore, 2001), page 1495 (arXiv:hep-ph/0010175).

[22] S. Eidelman *et al.* [Particle Data Group], Phys. Lett. B **592**, 1 (2004).

[23] M. Neubert, Phys. Rept. **245**, 259 (1994).

[24] S. L. Zhu and Y. B. Dai, Phys. Rev. D **59**, 114015 (1999).

[25] This expression does not coincide with that obtained in ref.[24].

[26] Y. B. Dai, C. S. Huang, C. Liu and S. L. Zhu, Phys. Rev. D **68**, 114011 (2003).

[27] P. Colangelo, F. De Fazio, G. Nardulli, N. Di Bartolomeo and R. Gatto, Phys. Rev. D **52**, 6422 (1995); P. Colangelo and F. De Fazio, Eur. Phys. J. C **4**, 503 (1998); P. Colangelo, F. De Fazio and N. Paver, Phys. Rev. D **58**, 116005 (1998).

[28] P. Ball, V. M. Braun and N. Kivel, Nucl. Phys. B **649**, 263 (2003).

[29] P. Colangelo, F. De Fazio, G. Nardulli and N. Paver, Phys. Lett. B **408**, 340 (1997).

[30] P. L. Cho and M. B. Wise, Phys. Rev. D **49**, 6228 (1994).